we can set it accurately at a distance from the centre equal to the eccentricity e.

(*b*) The pediometer is graduated from zero to unity, starting from the point A.

(*c*) For the revolving bar, we take a point on the side of the roller, at an unit distance from the centre. Starting from that point, we graduate towards the other side over the centre. Then, the reading of the point D (where the pediometer intersects the bar perpendicularly) gives directly the value of

$$1 - e \cos E.$$

Consequently, if we compute M_o, such that

$$M_o = E - e \sin E,$$

we can get readily the correction for the value of E so found, by the following expression,

$$\Delta E = \frac{M - M_o}{1 - e \cos E},$$

or $(M - M_o)$ divided by the reading of the point D.

Kyoto University Observatory:
1926 October 29.

Some Remarks on the Theory of Radiative Equilibrium in the Outer Layers of the Stars (in reference to the work of Professor E. A. Milne). By V. A. Ambarzumian and N. A. Kosirev.

§ 1. Professor E. A. Milne, in his very interesting paper "Radiative Equilibrium in the Outer Layers of a Star," published by him in 1921 March (*Monthly Notices of the R.A.S.*, pp. 361–375), derives the following equation, which characterizes the state of radiative equilibrium in the outer layers of stars, the generation of energy being absent,

$$2C'(\tau) = \int_0^\tau \frac{C(\tau + y) - C(\tau - y)}{y} e^{-y} dy + \int_\tau^\infty \frac{C(\tau + y)}{y} e^{-y} dy . \quad (1)$$

In the above equation

$$\tau = \int_0^x k\rho dx$$

where x is the height of the layer (the zero-point is taken on the surface), ρ the density, and k the coefficient of the absorption of the rays, which are going along the normal in the layer to be examined. Also,

$$C(\tau) = \int_0^\tau B(t)dt ; \qquad C'(\tau) = B(\tau) \quad . \quad . \quad (2)$$

where $B(\tau)$ represents the intensity of the radiation of an absolutely black body at the temperature of the given layer, this intensity being expressed by the law of Stefan. Solving the equation (1), we obtain the expression of the function $C'(\tau)$, and consequently also the relation between the temperature and τ. Professor Milne tries to find the solution of the equation (1) by means of the method of successive approximation. He assumes the first approximation to be

$$C'(\tau) = a + 2b\tau,$$

which corresponds to the distribution of temperature in layers infinitely removed from the surface. He stops at the second approximation, considering $C'(\tau)$ to be clearly expressed by the following formula:—

$$C'(\tau) = B(\tau) = a + 2b\tau + \tfrac{1}{2}e^{-\tau}(b - a - b\tau) + \tfrac{1}{2}(a\tau + b\tau^2)\int_\tau^\infty \frac{e^{-y}}{y}dy \quad (3)$$

thus assuming the above expression for an approximate solution of the equation (1). But in this question Professor Milne commits an error, as this equation has only one solution identically equal to zero,

$$C'(\tau) \equiv B(\tau) \equiv 0 \quad . \quad . \quad . \quad . \quad (4)$$

This indicates that radiative equilibrium without the generation of energy cannot take place if the intensity of radiation differs from zero.

In order to prove the relation (4), we shall show first that the solutions of the equation (1) will be at the same time the solutions of the following homogeneous integral equation

$$B(\tau) = \tfrac{1}{2}\int_0^\infty \mathrm{Ei}\,|\tau - t|\,B(t)dt \quad . \quad . \quad . \quad (5)$$

where we adopt the notation

$$\mathrm{Ei}x = \int_x^\infty \frac{e^{-\xi}}{\xi}d\xi.$$

This will be the case if the equation (5) follows from equation (1), which we are about to prove. Adding to and subtracting from the right-hand side of the equation the quantity

$$C(\tau)\int_\tau^\infty \frac{e^{-y}}{y}dy,$$

we find

$$2C'(\tau) = \int_0^\infty \frac{C(\tau + y) - C(\tau)}{y}e^{-y}dy - \int_0^\tau \frac{C(\tau - y) - C(\tau)}{y}e^{-y}dy + C(\tau)\int_\tau^\infty \frac{e^{-y}}{y}dy$$

Moreover we see that

$$\frac{C(\tau + y\mu)}{y} = \int C'(\tau + y\mu)d\mu \quad \text{and} \quad \frac{C(\tau - y\mu)}{y} = \int C'(\tau - y\mu)d\mu.$$

By these relations the subintegral functions in equation (6) are transformed into

$$\frac{C(\tau+y)-C(\tau)}{y} = \left[\frac{C(\tau+y\mu)}{y}\right]_0^1 = \int_0^1 C'(\tau+y\mu)d\mu$$

$$\frac{C(\tau-y)-C(\tau)}{y} = \left[\frac{C(\tau-y\mu)}{y}\right]_0^1 = -\int_0^1 C'(\tau-y\mu)d\mu$$

$$\frac{C(\tau)}{y} = \left[\frac{C(\tau-y\mu)}{y}\right]_{\tau/y}^0 = \int_{\tau/y}^0 C'(\tau-y\mu)d\mu$$

since from the first of the equations (2) it follows that C(0) = 0. Substituting in equation (6) the relations thus obtained, we find

$$2C'(\tau) = \int_0^\infty e^{-y}dy\int_0^1 C'(\tau+y\mu)d\mu + \int_0^\tau e^{-y}dy\int_0^1 C'(\tau-y\mu)d\mu + \int_\tau^\infty e^{-y}dy\int_0^{\tau/y} C'(\tau-y\mu)d\mu \quad (7)$$

The second term of the right-hand side of the equation is a double integral extended over the rectangle ($0 \leqslant \mu \leqslant 1$; $0 \leqslant y \leqslant \tau$) in the plane $y\mu$; the third member is a double integral with the same subintegral function, this integral being extended over a region which is limited by one side of the former rectangle and by $\tau = y\mu$. Thus we may replace these two integrals by one, extended over a new region composed of the two former. Altering the order of integration, equation (7) is transformed into

$$2C'(\tau) = \int_0^1 d\mu\int_0^\infty C'(\tau+y\mu)e^{-y}dy + \int_0^1 d\mu\int_0^{\tau/\mu} e^{-y}C'(\tau-y\mu)dy.$$

In the first integral, put

$$\tau + y\mu = t; \qquad dy = 1/\mu\, dt;$$

and in the second,

$$\tau - y\mu = t; \qquad dy = -1/\mu\, dt.$$

We obtain

$$2C'(\tau) = \int_0^1 d\mu\int_\tau^\infty C'(t)\frac{e^{-\frac{t-\tau}{\mu}}}{\mu}dt + \int_0^1 d\mu\int_0^\tau C'(t)\frac{e^{-\frac{\tau-t}{\mu}}}{\mu}dt.$$

Putting

$$1/\mu = \xi; \qquad d\mu = -\frac{d\xi}{\xi^2}$$

and, altering the order of integration, we find

$$2C'(\tau) = \int_\tau^\infty dt\int_1^\infty C'(t)\frac{e^{-(t-\tau)\xi}}{\xi}d\xi + \int_0^\tau dt\int_1^\infty C'(t)\frac{e^{-(\tau-t)\xi}}{\xi}d\xi,$$

but, as according to our notation,

$$Ei(t-\tau) = \int_{t-\tau}^\infty \frac{e^{-\xi}}{\xi}d\xi = \int_1^\infty \frac{e^{-(t-\tau)\xi}}{\xi}d\xi \quad \text{and} \quad Ei(\tau-t) = \int_1^\infty \frac{e^{-(\tau-t)\xi}}{\xi}d\xi,$$

we finally obtain

$$B(\tau) = \tfrac{1}{2}\int_0^\infty Ei\,|\tau - t|\,B(t)dt \quad . \quad . \quad . \quad (8)$$

because $B(t) = C'(t)$, which we had to prove. Now let us prove the statement that the homogeneous integral equation has not any continuous solutions which are not identically zero.

§ 2. It is readily calculated that

$$\tfrac{1}{2}\int_0^\infty Ei\,|\tau - t|\,dt = 1 - \tfrac{1}{2}(e^{-\tau} - \tau Ei\tau) \quad . \quad . \quad (9)$$

Let us show that in the whole region $0 \leqslant \tau < \infty$ we have

$$\tfrac{1}{2}(e^{-\tau} - \tau Ei\tau) > 0 \quad \text{or} \quad e^{-\tau} > \tau Ei\tau \quad . \quad . \quad . \quad (10)$$

In fact

$$e^{-\tau} = \int_\tau^\infty e^{-\xi} d\xi; \qquad \tau Ei\tau = \tau\int_\tau^\infty \frac{e^{-\xi}}{\xi} d\xi = \int_\tau^\infty \frac{\tau}{\xi} e^{-\xi} d\xi.$$

As in both integrals the subintegral functions are positive, and as the second function differs from the first by the factor $\frac{\tau}{\xi} \leqslant 1$, the inequality is proved.

Let us denote the positive function $\frac{1}{2}(e^{-\tau} - \tau Ei\tau)$ by $\psi_1(\tau)$. We may write

$$\tfrac{1}{2}\int_0^\infty Ei\,|\tau - t|\,dt = 1 - \psi_1(\tau) = \phi_1(\tau) \quad . \quad . \quad (11)$$

where $\phi_1(\tau)$ is positive, as $Ei\,|\tau - t|$ is positive on the whole plane τt. We form an infinite sequence of functions connected together by the relation

$$\tfrac{1}{2}\int_0^\infty Ei\,|\tau - t|\,\phi_{n-1}(t)dt = \phi_n(t) \quad . \quad . \quad . \quad (12)$$

All the functions of this sequence are positive since the kernel $Ei\,|\tau - t|$ and the function $\phi_1(\tau)$ are positive. Thus $\phi_n(\tau) > 0$. We had $\phi_1(\tau) = 1 - \psi_1(\tau)$, where $\psi_1(\tau)$ is a positive function. If we substitute this equation in (12) we obtain, having in view (11),

$$\phi_2(\tau) = 1 - \psi_1(\tau) - \psi_2(\tau)$$

where

$$\psi_2(\tau) = \tfrac{1}{2}\int_0^\infty Ei\,|\tau - t|\,\psi_1(t)dt$$

is also a positive function. Continuing it, we obtain for $\phi_n(\tau)$ the formula

$$\phi_n(\tau) = 1 - [\psi_1(\tau) + \psi_2(\tau) + \ . \ . \ . + \psi_n(\tau)] > 0 \quad . \quad (13)$$

where

$$\psi_n(\tau) = \tfrac{1}{2}\int_0^\infty Ei\,|\tau - t|\,\psi_{n-1}(t)dt.$$

Evidently the sequence of the functions $\phi_n(\tau)$ converges. In fact at every value of τ this sequence is monotonically decreasing, but all its members are still greater than zero. Let us denote the limit of this series by $\Phi(\tau)$. Let us prove that our sequence of functions uniformly converges. For this purpose let us first show that out of it one may extract a uniformly converging series.

Our sequence of functions satisfies the conditions which have been formulated by Courant and Hilbert as sufficient to make possible the extraction of a uniformly converging sequence.* First of all it is uniformly limited because all $|\phi_n(\tau)| < 1$. Secondly it is uniformly continuous because

$$\phi_n(\tau+\eta) - \phi_n(\tau) = \tfrac{1}{2}\int_0^\infty [\mathrm{E}i|\tau+\eta-t| - \mathrm{E}i\,|\tau-t|\,]\phi_{n-1}(t)dt$$
$$= \phi_{n-1}(\bar{t})[\phi_1(\tau+\eta) - \phi_1(\tau)]$$

where the factor $\phi_{n-1}(\bar{t})$ is taken outside the sign of integration, according to the theorem of the mean. Since all $\phi_{n-1}(\bar{t}) < 1$ from the uniform continuity of $\phi_1(\tau)$ there follows the uniform continuity of the whole sequence $\phi_n(\tau)$. Thus the sequence $\phi_n(\tau)$ is uniformly limited and uniformly continuous, consequently we are able to extract from it, according to the theorem of Courant and Hilbert, a uniformly converging sequence. It is obvious that the limit of this partial sequence will be the same as the limit of the whole sequence, viz. $\Phi(\tau)$. Let us show that the whole sequence as well converges uniformly. In fact any sufficiently far term of the whole sequence is enclosed between two terms ϕ_{n_i} and $\phi_{n_{i+1}}$ of the partial sequence. Since for all the positive τ we have

$$|\Phi(\tau) - \phi_{n_i}(\tau)| < \epsilon \qquad |\Phi(\tau) - \phi_{n_{i+1}}(\tau)| < \epsilon \quad . \quad . \quad (14)$$

so by virtue of the monotonity of the whole sequence for n, which is enclosed between n_i and n_{i+1}, we have also

$$|\Phi(\tau) - \phi_n(\tau)| < \epsilon \quad . \quad . \quad . \quad . \quad (15)$$

Thus the series of functions

$$\phi_1(\tau), \quad \phi_2(\tau), \quad \ldots, \quad \phi_n(\tau), \quad \ldots$$

converges uniformly for all positive values τ. Consequently also the sequence of functions

$$1-\phi_1(\tau), \quad 1-\phi_2(\tau), \quad \ldots, \quad 1-\phi_n(\tau), \quad \ldots$$

converges uniformly for all the values τ. As

$$1-\phi_n(\tau) = \psi_1(\tau) + \psi_2(\tau) + \ldots + \psi_n(\tau)$$

the series $\psi_1(\tau) + \psi_2(\tau) + \ldots$ converges uniformly, all τ being positive. Let us show that this series if multiplied by $\mathrm{E}i\,|\tau - t|$ admits an integration term by term in the whole interval, that means, if we put

$$\Psi(\tau) = \psi_1(\tau) + \psi_2(\tau) + \ldots + \psi_n(\tau) + \ldots \quad . \quad (16)$$

* R. Courant und D. Hilbert, *Methoden der Mathematischen Physik*, I, pp. 39–41.

then

$$\int_0^\infty \mathrm{E}i\,|\tau - t|\,\Psi(t)dt = \int_0^\infty \mathrm{E}i\,|\tau - t|\,\psi_1(t)dt + \int_0^\infty \mathrm{E}i\,|\tau - t|\,\psi_2(t)dt + \;.\;\;.\;\;. \quad (17$$

The integral on the left indubitably exists, since $\Psi(\tau)$ is continuous, and is limited, and $\mathrm{E}i\,|\tau - t|$ is integrable from zero to infinity. In virtue of the uniform convergence of the series (16), the integration term by term is possible between finite limits

$$\int_0^b \mathrm{E}i\,|\tau - t|\,\Psi(t)dt = \int_0^b \mathrm{E}i\,|\tau - t|\,\psi_1(t)dt + \int_0^b \mathrm{E}i\,|\tau - t|\,\psi_2(t)dt + \;.\;\;.\;\;.$$

where b may be no matter how great, and therefore the left part will differ no matter how little from

$$\int_0^\infty \mathrm{E}i\,|\tau - t|\,\Psi(t)dt\,;$$

consequently also the right part will differ from the same integral no matter how little and thus, by increasing "b", will tend to it as a limit.

The possibility of an integration term by term of the series being proved,

$$\mathrm{E}i\,|\tau - t|\,\Psi(t) = \mathrm{E}i\,|\tau - t|\,\psi_1(t) + \mathrm{E}i\,|\tau - t|\,\psi_2(t) + \;.\;\;.\;\;. \quad (18)$$

we find the solution of the following integral equation :—

$$\psi_1(\tau) = \phi(\tau) - \tfrac{1}{2}\int_0^\infty \mathrm{E}i\,|\tau - t|\,\phi(t)dt \quad . \quad . \quad (19)$$

in the form

$$\phi(\tau) = \Psi(\tau) = \psi_1(\tau) + \psi_2(\tau) + \;.\;\;.\;\;. + \psi_n(\tau) + \;.\;\;.\;\;. \quad (20)$$

It is easy to convince oneself of this by substituting this series in (19) and taking into consideration that

$$\psi_n(\tau) = \tfrac{1}{2}\int_0^\infty \mathrm{E}i\,|\tau - t|\,\psi_{n-1}(t)dt.$$

Thus the equation (19) has a thoroughly determinate solution $\Psi(\tau)$. But if the homogeneous equation

$$\mathrm{B}(\tau) = \tfrac{1}{2}\int_0^\infty \mathrm{E}i\,|\tau - t|\,\mathrm{B}(t)dt \quad . \quad . \quad . \quad (21)$$

should have a solution not identically equal to zero, then

$$\int_0^\infty \mathrm{B}(\tau)\psi_1(\tau)d\tau$$

should be equal to zero, as, if we multiply (19) by $\mathrm{B}(\tau)$, we obtain

$$\int_0^\infty \psi_1(\tau)\mathrm{B}(\tau)d\tau = \int_0^\infty \phi(\tau)\mathrm{B}(\tau)d\tau - \tfrac{1}{2}\int_0^\infty \mathrm{B}(\tau)d\tau\int_0^\infty \mathrm{E}i\,|\tau - t|\,\phi(t)dt \quad (22)$$

whence, taking into consideration (21) and altering the order of integration, we obtain

$$\int_0^\infty B(\tau)\psi_1(\tau)d\tau = \int_0^\infty \phi(\tau)B(\tau)d\tau - \int_0^\infty \phi(t)B(t)dt = 0 \quad . \quad (23)$$

Thus the functions $B(\tau)$ and $\psi_1(\tau)$ must be orthogonal. As $\psi_1(\tau)$ is positive, the function $B(\tau)$ must have a negative value (and therefore without physical significance). Let us form a new continuous positive function $\chi(\tau)$, which differs from $\psi_1(\tau)$ in those regions where $B(\tau)$ is negative, $\chi(\tau)<\psi_1(\tau)$, and in other regions $\chi(\tau) = \psi_1(\tau)$. It is obvious that this function does not satisfy the condition of orthogonality of

$$\int_0^\infty \chi(\tau)B(\tau)d\tau = 0.$$

Meanwhile the equation

$$\chi(\tau) = \phi(\tau) - \tfrac{1}{2}\int_0^\infty \mathrm{E}i\,|\tau - t|\,\phi(t)dt$$

has a thoroughly determinate solution. Therefore in case there exists a solution (21) which cannot be identically equal to zero, the condition of orthogonality should take place, as it may be shown analogically to the above mentioned.

Thus, assuming the existence of a solution of the equation (21) which cannot be identically equal to zero, we have come to a contradiction. Such a solution does not exist, consequently there does not exist any continuous solution (1) different from zero.

Therefore the method used by Professor Milne is not applicable in this case.

Micrometrical Measures of Double Stars (21*st Series*).

By Rev. T. E. Espin and W. Milburn.

The Micrometrical Measures are given as in previous lists, those marked E being made with the 24-inch and the ones marked M with the 17½-inch. The asterisk attached to a star in Column 1 denotes that additional information is given in the notes.

Name.	R.A. (1900)	Decl. (1900)	P.	D.	Mags.		Nts.	Date 1926.	
	h m	° ′	°	″					
Espin 312	0 12·0	+34 35	237·0	2·47	9·5	9·7	3	·283	E
h 622	20·7	34 14	131·1	19·44	9·0	9·2	2	·030	E
Espin 314	30·6	28 41	200·2	7·93	9·0	14·0	2	·980	M
								1927.	
M 184 *	37·8	63 39	238·4	2·79	10·5	12·0	3	·005	M BC
			63·7	6·97	A = 10·0		2	·009	M AB